Quine on Identity

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Abstract

In a first section, we discuss Quine's claim according to which identity is a logical notion. We point out that Quine mixes up various types of identities: trivial (or diagonal) identity, Leibniz identity, etc and this leads him to commit several mistakes.

In a second section, we review Quine's criticisms to various philosophers (Wittgenstein, Whitehead, Leibniz, etc), who according to him made a confusion between name an object in defining identity. We show that in fact only Korzybski can be accused of such confusion.

In a third section, we analyze the relation between identity and entity. We notice that for Quine a river is the result of the identification of river stages, but that he admits it as an entity by opposition to squareness which according to him is a result of an identification process of higher abstraction.

Keywords : Quine, identity, definissability of identity, entity, singular terms

0. So Simple?

At different successive stages of the development of his thought, Quine reiterates the same affirmation according to which identity is very simple but people have not been able to properly understand it (we will see later on that these people, according Quine, include famous philosophers like Heraclitus, Leibniz, Wittgenstein and Whitehead) :

- 1942: "Embora seja a identitade uma noção tão elementar, tem sido objeto de confusões persistentes", *O sentido da nova lógica*[2] (SNV, §32, p.135).
- 1950: "Despites its simplicity, identity invites confusion", *Methods of logic*, (MOL §40, p.221).
- 1960: "Though the notion of identity is so simple, confusion over it is not uncommon", *Word and object*, (W&O, §24 p.117).

In what follows, we will see that Quine in fact makes several confusions over identity, so that we could say: "Though the notion of identity is so simple, Quine

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[2] Quine wrote this book in Portuguese when he was in Brazil in 1941-42. This book was never translated in English. We may think it is because most of its content has been rewritten in English by Quine in several papers and books. Anyway, in this book Quine had the opportunity to put most of his philosophical ideas in an organized whole in a very short time, when he was still quite young (see Quine 1997). And it is interesting to compare this book with *Philosopy ofl logic* he wrote 30 years later. So the book is valuable for Quineans, neo-Quineans, post-Quineans, anti-Quineans.

makes confusion over it" and add Quine to the above list of celebrities. But this would not be a faithful description of our view. Our opinion is that identity is not so simple. Our motto is rather "Despites its apparent simplicity, identity is not so easy to deal with". According to us, the main mistake of Quine is not to have perceived the complexity of identity and to have ignored and misinterpreted several important technical results about this notion.

1. Is Identity Logical?

In *Philosophy of logic*, Quine sustains that identity has the four following features (POL, Chapter 5, pp.61-65):

(c) Identity is complete
(e) Identity is definable/eliminable
(d) Identity is uniquely determined
(u) Identity is universal

For him, these are four good reasons to conclude that:

(l) Identity is logical

To assess Quine's claims we need to recall a few facts about identity. As it is known, identity is not model-theoretically axiomatizable in first-order logic, in the same sense, e.g. that the notion of well-order is not axiomatizable: it is not possible to find a recursive set of axioms about a binary relation = such that in the models of these axioms = is always identity (see e.g. Hodges 1983).

Despite this result, many people still think that Leibniz's schema

$$\text{(L)} \qquad \forall x \forall y\, (x{=}y \leftrightarrow (\varphi x \leftrightarrow \varphi y))$$

is an model-theoritical first-order axiomatization of identity. However Leibniz's schema axiomatizes a congruence relation, namely the Leibniz congruence, which most of the time is not identity. Of course one may call Leibniz congruence, identity, and states that identity is axiomatizable. In the same way, one can call the Pope, God, and state that God exists. However if we call identity identity, identity is not axiomatizable. By identity we mean the relation according to which every element is related to itself and nothing else, to avoid any confusion we will call this relation *trivial identity*.

It is reasonable to think that Leibniz congruence, and other relations, can rightly be called identity relations, but we must then be very careful when using the word "identity". We have to leave no space for ambiguity, the reader should be able to have obvious answers to the questions: are we talking to a specific identity (trivial identity, Leibniz congruence, something else) and which one of these? Or are we talking of all possible relations of identity?

A serious defect of Quine (and other philosophers)'s gloss on identity is the lack of this kind of specifications.

These distinctions made, it is clear that contrarily to what Quine says, identity is not definable and eliminable by his method in the context of a language with a finite number of predicates. This has already been pointed out by various people (see e.g. Savellos, 1990).

After giving an example of his method in the case of four predicates, which consists in writing a formula called (3) corresponding to the Leibniz's schema for atomic formulas, Quine says: "It may happen that the objects intended as values of the variables of quantification are not completely distinguishable from one another by the four predicates." (POL, p.63) And he makes the following comments: "When this happens, (3) fails to define genuine identity. Still, such failure remains unobservable from within the language; (3) is as good as identity from that vantage point." (POL, p.63).

The formula (3) in fact describes Leibniz congruence, we can call it an identity, Leibniz identity, but the important point is that it is not trivial identity, "genuine identity" to use Quine's words. So what is definable and eliminable is Leibniz identity, not trivial identity, or identity in general. The same can be said about the alleged uniqueness of determination of identity. What is uniquely determined is Leibniz identity. On the other hand, the non axiomatizability of trivial identity expresses the impossibility of univocal determination of identity.

Furthermore, to say that Leibniz identity is as good as trivial identity, since the difference between the two does not manifest in the language would be the same as saying that Champagne is as good as Coca-Cola for the Matutos of Brazil, since in their language they have only one word, "Torakata", for any sparkling drink.

It is not possible to conclude that trivial identity is logical from (e) or (d) since (e) or (d) does not hold for trivial identity, all we know is that (e) and (d) hold for Leibniz identity. Is it enough to call Leibniz identity logical?

Anyway, let us see now if (c) or (u) holds for identity (trivial or not).

Quine writes: "Another respect in which identity theory seems more like logic than mathematics is universality: it treats of all objects impartially. Any theory can indeed likewise be formulated with general variables, ranging over everything, but still the only values of the variables that matter to number theory, for instance, or set theory, are the numbers and the sets; whereas identity theory knows no preference" (POL, p.62).

The basic idea beyond this statement is correct, but the idea is expressed in a very unclear and confused way. What does mean "the only values of the variables that matter to number theory are numbers"? Given Peano axioms, the value of the variables are any objects of a model of these axioms. And many of these objects are not numbers in the usual sense of the word. In fact what Quine says here is totally contrary to the modern conception of axiomatization developed by Hilbert according to which for example in the axiom of geometry “point” and “line” can be interpreted by “glass of beer” and “table”.

What is true is that given any set, we can define trivial identity: it is the diagonal of the Cartesian product. So identity is a relation that can be defined on any set of objects. This is a good reason to consider it as universal.

Note that identity is not the only binary relation universal in this sense: there are other ones, in particular the three other binary relations which are invariant under any one-to-one correspondence are also universal. These notions are diversity (the negation of identity), the empty relation and its negation (the universal relation). In a famous 1966 conference, Tarski defines theses four notions as “logical” for this reason, and he claims that “these are the only logical binary relations between individuals.” (p.150)[3].

What Tarski is talking about here is trivial identity. It is easy to see in fact that Leibniz identity is not T-logical (logical in the sense of Tarski, i.e. invariant under arbitrary transformations). On the other hand Leibniz identity is universal, it can be defined on any set of object. So we have universal notions which are not T-logical. It is not clear that Quine reached this kind of subtlety. Anyway, the fact that Leibniz identity is not T-logical maybe use against its logicality.

Quine sustains also that a good reason to consider identity as logical is that it is proof-theoretically axiomatizable in first-order logic as proved by Gödel with his completeness theorem.

This completeness theorem is proved relatively to a semantics in which identity is considered as a logical constant which is always interpreted in the models as trivial identity.

Quine says: “Elementary number theory, in contrast, is shown in Gödel’s more famous theorem (1931) to admit no complete proof procedure” (POL, p.62). Note that this is not in fact only number theory, but any non universal theory that cannot be proof-theoretically axiomatizable in the same way as identity theory, because to provide such an axiomatization, we must have a semantics in which the relations of the theory can be considered as logical constants, in particular can be defined in any models. On the other hand the T-

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[3] Although Tarski’s consideration goes back to a result he proved with Lindenbaum in 1936 it is still not very well-known and rarely presented and commented in books despite his recommendation: “Though this result is simple, I think that it should be included in most logical textbooks.” (1986, p.150).

Tarski also recalls that “just these four relations were introduced and discussed in the theory of relations by Peirce, Schröder and other logicians of the nineteenth century” (1986, p.150).

logical notions can also be proof-theoretically axiomatized in this way. For example the universal relation can be very easily axiomatized by the axiom: $\forall x \forall y\ (xRy)$. As we have seen, Leibniz identity is also universal. Gödel's axiomatization can in fact be considered as an axiomatization of Leibniz identiy.

If we call Q-logical, a notion obeying (c), (e), (d), (u), trivial identity is not Q-logical since it does not obey (e) and (d). On the other hand, we have seen that Leibniz identity is not T-logical. It seems that the universal and the empty relations are the only notions which are both T and Q-logical, which therefore can be called logical if we want to conciliate Quine and Tarski and define logical as Q-logical and T-logical.

2. Is Identity a Relation between Signs or between Objects?

Commenting various authors, Quine says that "the root of this trouble (about identity) is confusion of sign and object" (W&O, p.116). Here is his comments on the Austrian logico-philosophicus:

> Wittgenstein's mistake is more clearly recognizable, when he objects to the notion of identity that "to say of *two* things that they are identical is nonsense, and to say of *one* thing that it is identical with itself is to say nothing". Actually of course the statements of identity that are true and not idle consist of unlike singular terms that refer to the same thing (W&O, p.117).

In fact there are non idle statements of identity that do not consist of unlike singular terms that refer to the same thing, that are not statements between different signs referring to the same object, but are between *different* objects being identified.

A simple example is, according Leibniz's principle of indistinguishability of indiscernibles, the identification of indistinguishable objects. Another example is given by the axiom of extensionality in set theory: in this case one identifies two sets when they have the same elements. The principle of abstraction is still another example, but a bit different: we gather together different objects having a common property: this is basically the process of conceptualization on which thought and language are based. When I say that *Minou* and *Minette* are of the *same* species, that they are both cats, I put forward a common feature, making abstraction of their differences.

These processes are different from the identification of singular terms denoting the same object, such as Isidor Ducasse and the Author of Maldoror Songs, the Morning Star and the Evening Star, 2+3 and 3+2. What are here identified are not signs, nor the denotation signs, but the meaning of signs. Two meanings are identified because they refer to the same object. In this case identity can be seen as a congruence relation between meanings, with respect to reference; this is the point of view of the extentionalist.

Quine has also no mercy for Leibniz:

> Similar confusion of sign and object is evident in Leibniz where he explains identity as a relation between the signs, rather than between the named object and itself: *Eadem sunt quorum unum potest susbtitui alteri, salva veritate*. (W&O, p.116)".

In fact, there is not necessarily a confusion here: in the same way that we can identify two signs congruent with respect to reference (extensionalism), we can identify two signs congruent with respect to truth (veritatism). These signs may have different meanings, they may also have different references.

Frege in his famous 1892's paper didn't reach this level of subtlety and defend an extensionalistico-veritatistic view of propositions soi-disant based on Leibniz's salva veritate conception of identity.

Quine also criticizes his PhD advisor, A.N.Whitehead:

> Identity evidently invites confusion between sign and object in men who would not make the confusion in other contexts. Those involved include most of the mathematicians who have liked to look upon equations as relating numbers that are somehow equal but distinct. Whitehead once defended the view, writing e.g. that "2+3 and 3+2 are not identical; the order of the symbols is different in the two combinations, and this difference of order directs different processes of thought. (W&O, p.117).

But if Whitehead considers that 2+3 and 3+2 are not identical because the difference of symbols reflects different processes of thought, he is not making the alleged confusion between sign and object. The processes of thought can be interpreted as the meaning of the symbols and Whitehead as a non extensionalist.

This is a quite different position as the Korzybski's one: "he argues that 1=1 must be false because the two sides of the equation are spatially distinct" (W&O, p.117). It seems that this is the only case presented by Quine where there is a real confusion between sign and object. We can in fact wonder how Quine could have imagined that Wittgenstein, Whitehead and Leibniz were as bad as Prince Korzybski.

Identity is a relation between objects not between sings, except when signs are taken as particular cases of objects. To correctly expresses his view Korzybski should have said: "1"= "1" is false.

Wittgenstein writes in his *Tractactus:* "Gleicheit des Gegenständes drücke ich durch Gleichheit des Zeichens aus, und nicht mit Hilfe eines Gleichheitszeichens. Verschiedenheit der Gegenstände durch Verschiedenheit der Zeichen" (5.53). So he will not write, $a=a$, because it is trivial, nor $a=b$, because it is false.

Quine says: “Of what use is the notion of identity if identifying an object with itself is trivial and identifying it with anything else is fales? This particular confusion is cleared up by reflecting that there are really not just two kinds of cases to consider, one trivial and the other false, but three:

Cicero=Cicero, Cicero=Catilines, Cicero=Tully.

The first of these is trivial and the second false, but the third is neither trivial nor false. The third is informative, because it joins two different terms; and at the same time it is true, because the two terms are names of the same objects. (…) it is not the names that are affirmed to be identical, it is the things named. Cicero is identical with Tully (same man), even though the name ‘Cicero’ is different from the name ‘Tully’” (MOL, §40, p.221).

One may wonder: why should we have different names for naming the same thing?

Quine’s analysis of the third case seems wrong: we don’t have two different names for the same thing. We are identifying two different things, and because these things are different, they have different names. Language generally respects the second half of Wittgenstein’s principle: two different names name different things. They may have the same reference, but this is another point. Quine analysis is based on a abuse of Frege’s distinction between *Sinn* und *Bedeutung*. Most philosophers make this kind of abuse, which consists in saying that the name names the reference, instead of saying that the name refers to the reference.

If we say: Paris=Lutèce, we establish a relation between two different things, two different cities. We are saying that in some sense they can be identified.

On the other hand, natural language generally does not respect the first half of Wittgenstein’s principle. We use the same name for different things, we say: “I will be in Paris tomorrow” and not “I will be in the Paris of tomorrow tomorrow”. We say also ambiguously: “Paris was called Lutèce”.

The functioning of language and thought is in fact based on the lack of respect of the first half of this principle.

3. Can we have Entities without Identity?

The slogan “No entity without identity” is one of two udders of Quine’s philosophy, the other one being “To be is to be the value of a variable”. If Quine will be remembered at all, it will be for these two commandments. Both are easy to remember and kick our minds, but what do they really mean?

The second commandment is not part of our agenda here and anyway, we have dealt with it elsewhere (2004b). The first is related to identity, topic of our present interest. Quine sustains that only entities for which we have a criterion of identity are real entities. Applying this commandment to the debate on propositions, he argues that propositions are not entities, because according to him, we don't have a criterion of identity for them.

But of what kind of identity are we talking about? Probably not trivial identity: we don't need no criterion to identify an object with itself. If it is not trivial identity, it is Leibniz identity or some other identity relations.

Quine rejects propositions but defends sentences. What is a sentence and do we have a citerion of identity for sentences? As Lesniewski pointed out long time ago, in logic we deal not with inscriptions but with sets of equiform inscriptions. A class of equiform inscriptions is a sentence. Equiformity can be considered as a criterion of identity for inscriptions. In this case inscriptions are entities. But can we infer from this that we have a criterion of identity for sentences and that sentences are entities? It seems that's what Quine is doing, but this inference is wrong: sentences are sets and to identify two sets, we need to use furthermore the principle of extensionality.

The fact that equiformity is a relation of equivalence is not necessarily obvious, on the contrary it seems that transitivity does not hold. In fact Lesniewski *required* equiformity to be transitive (1938, §11). Now, is equiformity a congruence relation? Congruency means compatibility (Bourbaki), but compatible with what? It all depends on the context, on the structure. Equiformity considered as a congruence relation means that we do not care about the material form of the inscription, so what do we care about? In logic, we may say that we care about logical form, this leads in fact to a further identification process, where we consider schemes of formulas (this kind of considerations led H.B.Curry to combinatory logics). Another possibility is to identify logically equivalent sentences, this leads to Lindenbaum-Tarksi algebra and algebraic logic.

The fact is that we may consider many different congruent relations between inscriptions. And the same hold for any class of entities. This has been pointed out by Suppes (1986). He took the case of spoken language, explaining how one can define a whole hierarchy of congruence relations between sound waves. So there is not only one concept of propositions (or sentences), but many.

All the entities we are dealing with in language and thought can be considered as the result of a simplification: equivalent class of objects under a congruent relation. This means in particular that these entities are not real, but they are an abstraction, a simplification of reality.

The question therefore is not whether they are real or not, but if we need them or not. This point of view is not necessarily different from Quine's. When commenting on the introduction of further abstractions, he writes, "our standard for appraising basic changes of conceptual scheme must be, not a realistic standard of correspondence to reality, but a pragmatic standard." (IOH, p.632)

Nonetheless the way Quine speaks about ontology and entities is sometimes ambiguous. For example, one would imagine that according to him, sentences exist but not propositions. But by saying that sentences are entities, he is not saying that they are part of an "unconceptualized reality" (IOH, p.632). On the other hand when he doesn't want to admit propositions in his "ontology", it is because he is not willing, without necessity, to perform a further step into abstraction: admitting abstractions of abstractions.

Quine says about Heraclitus famous river: "The truth is that you *can* bathe in the same *river* twice, but not in the same river stage" (IOH, p.621). He doesn't agree with Hume, according to whom, "the idea of external objects arises from an error of identification" (IOH, p.622). He thinks that in the case of the river "the imputation of identity is essential, to fixing the reference of the ostension" (IOH, p.622). So the river is constructed by identifying various river stages. He is ready to admit the existence of the river, river in which he can bathe as much as he wants. On the other hand he is not ready to admit the existence of the reference of general terms, like redness and squareness, although he is not rejecting general terms themselves.

But all this is not so obvious: we can *say* that we bathe in the same river twice and it makes sense, but can we really *do* it: bathe in the same river twice. If we can bathe in a river by bathing in a river stage, why cannot we draw squareness on the blackboard by drawing a square on it?

The idea beyond the metaphorical Heraclitus' river is that of universal flux (catharsis): everything change all the time, there is nothing permanent. We can argue that if we cannot bathe twice in the same river it is because the river itself does not exist. Some people have tried to seriously defend this point. This is for example the case of the physicist David Bohm, who have tried to rethink all modern physics from this viewpoint, with his theory of holomovement (1980).

Quine is rather interested in making a distinction between the river and squareness, than radically claiming that there is no river, although he may think so. His negative emphasis is on squareness. By opposition the river seems almost real.

But if we basically rely on a criterion of identity for the "existence" of "entities", it seems much easier to find a criterion of identity for abstract entities. For example, it is very easy to identify odd numbers: since we have a mechanical process that we can use to check if two given numbers are the same, in the sense that both are odd. This is much easier than to identify two concrete objects such as sound waves or handwritings.

It seems therefore that the commandment "No entity without identity" has an effect opposed to the one expected by Quine: oddity is easily identified, but this not necessarily the case of more concrete things like smells, feels, timbres, etc. as Strawson (1976) has pointed out. In these cases we may be able to perform the identification, but we are not able to state precisely a criterion of identity.

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